III. N.D. with Central Limit Theorem $\leftarrow$ for sample $\bar{x}$

Central Limit Theorem

Suppose we have a population with size N with $x_1, x_2, \ldots, x_n$

Then,
$$\mu = \frac{\sum x}{N}$$
$$\sigma^2 = \frac{\sum (x-\mu)^2}{N}$$
$$\sigma = \sqrt{\frac{\sum (x-\mu)^2}{N}}$$

Now, if $\bar{x}$ is the sample mean of a random size n from the population size N, then we have

$$\boxed{\text{- mean of } \bar{x} : \mu} \quad \leftarrow \quad \frac{\bar{x}_1 + \bar{x}_2 + \cdots + \bar{x}_n}{n} \approx \bar{x}$$

$$\boxed{\text{- Standard deviation of } \bar{x} : \frac{\sigma}{\sqrt{n}}} \quad \leftarrow \quad \text{Var}: \frac{\sum (x-\mu)^2}{N}$$

$$\text{Std. dev}: \sqrt{\frac{\sum (x-\mu)^2}{N}} = \sqrt{\frac{\sigma^2}{N}} = \frac{\sigma}{\sqrt{n}}$$

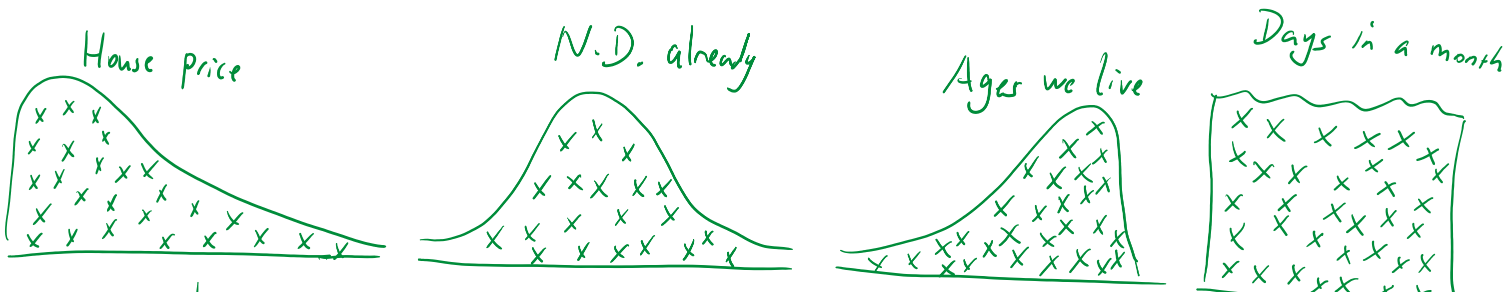
Therefore, the Central Limit Theorem implies if we take a random sample size n ($n > 30$) from a population size N. Then $\bar{x}$ has a new mean μ and a new standard deviation $\frac{\sigma}{\sqrt{n}}$.

The formula with sample $\bar{x}$ is:

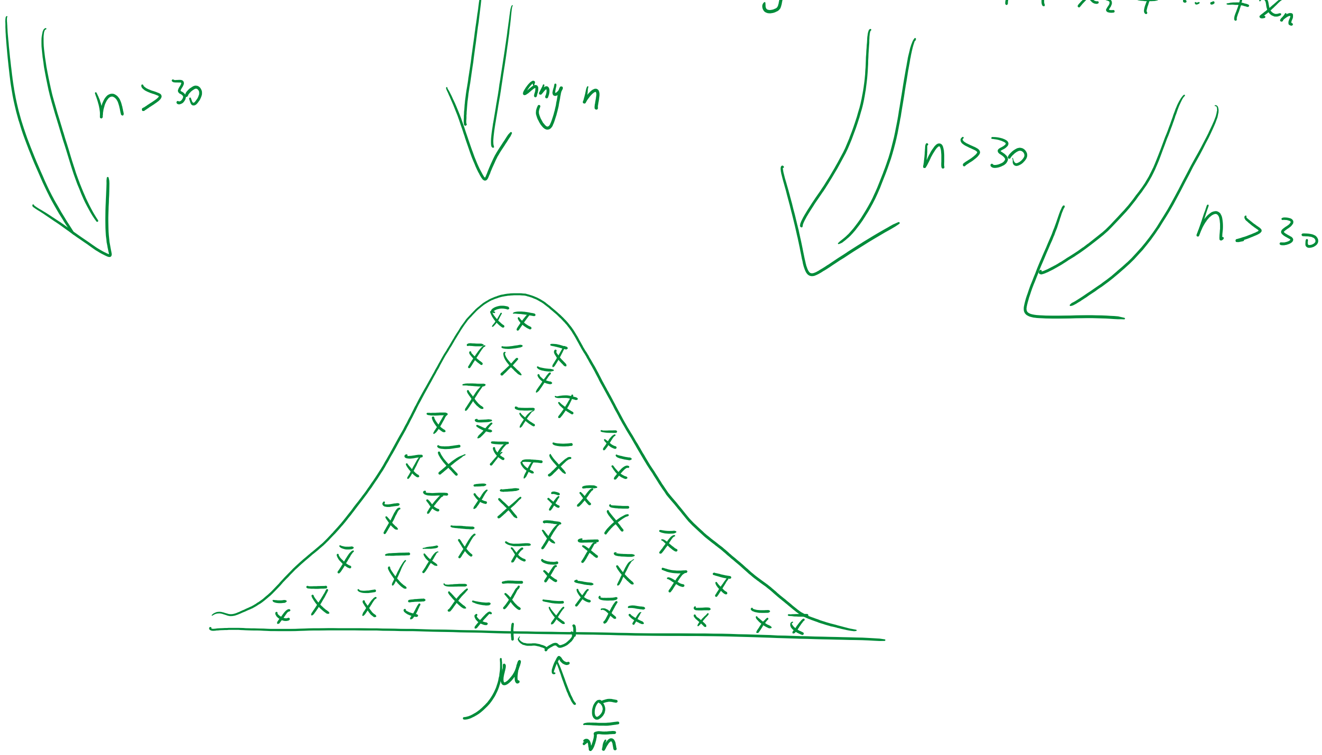
$$z = \frac{\bar{x} - \mu}{}$$

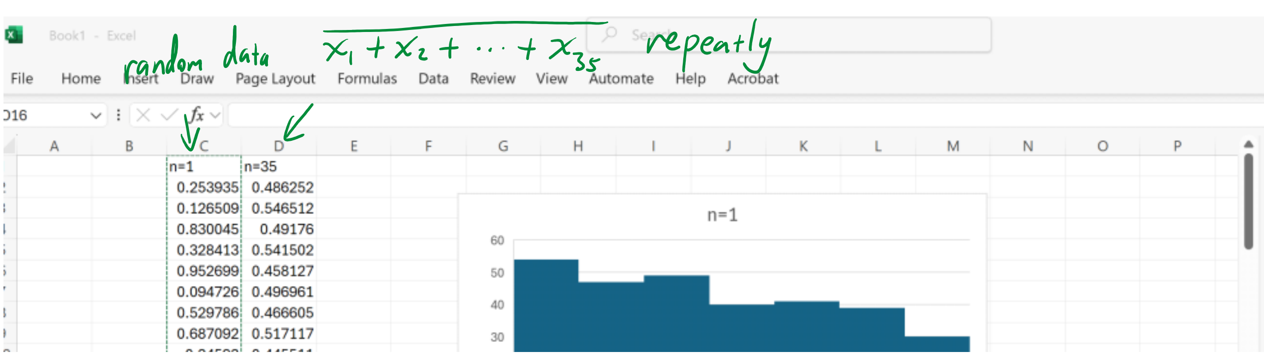
$$\boxed{z_{\bar{x}} = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}}$$

Graphic approaches:



take groups of $n > 30$ sample size's average $\bar{x}_n$ "$\overline{x_1 + x_2 + \cdots + x_n}$"

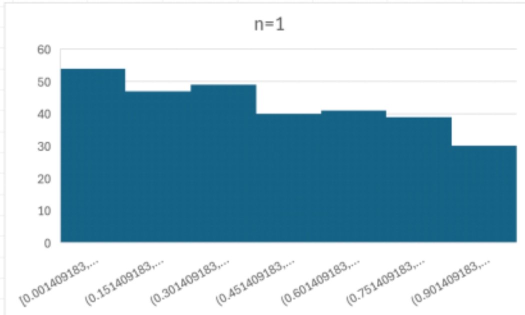


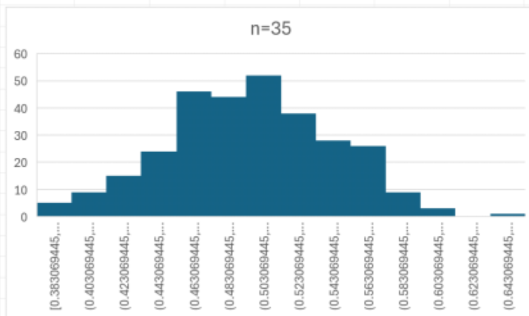


File Home Insert Draw Page Layout Formulas Data Review View Automate Help Acrobat

random data $\quad x_1 + x_2 + \cdots + x_{35}$ repeatly

n=1	n=35
0.253935	0.486252
0.126509	0.546512
0.830045	0.49176
0.328413	0.541502
0.952699	0.458127
0.094726	0.496961
0.529786	0.466605
0.687092	0.517117
0.24592	0.445511
0.337015	0.524711
0.360236	0.55375
0.009324	0.513381
0.103846	0.466546
0.187578	0.580387
0.314464	0.492757
0.440602	0.495864
0.343255	0.465323
0.711786	0.513748
0.462868	0.558925
0.371351	0.553445
0.250378	0.540514
0.498233	0.469432
0.644945	0.471294
0.015918	0.457494
0.961334	0.581317
0.42689	0.522083
0.408988	0.620795
0.692263	0.506806
0.818449	0.599841
0.409266	0.383069
0.350491	0.460313
0.21715	0.517519
0.042376	0.485724
0.990591	0.507254
0.077826	0.512456
0.829365	0.520059
0.311513	0.547517





Sheet1

Why? It is the nature! (eg Snowflakes has 6 sides.)

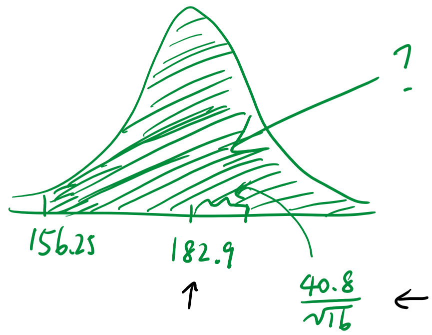
Thus, everything could turn into a bell-curve.

eg Suppose an elevator has a maximum capacity of passengers with a total weight of 3000 lb. Assume male weights follow a normal distribution with a mean of 182.9 lb and a standard deviation of 40.8 lb. Find the probability that a sample of 16 males have a mean weight greater than 156.25 lb.

S: $\quad n=16$ $\quad$ Here it means $\quad$?

$$\frac{x_1 + x_2 + \cdots + x_{16}}{16}, \text{ repeatly}$$

$$\frac{x_1 + x_2 + \cdots + x_{16}}{16}, \quad \underline{\text{repeatly}}$$

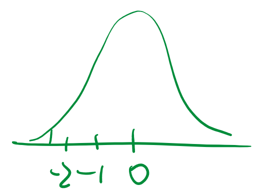


$$P(\bar{x} > 156.25) = 1 - P(\bar{x} < 156.25)$$

Now, $$z_{\bar{x}} = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$= \frac{156.25 - 182.9}{\frac{40.8}{\sqrt{16}}}$$

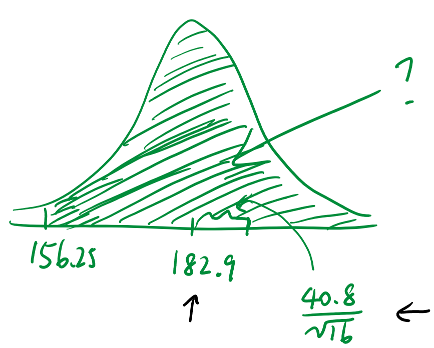
$$\approx -2.61$$

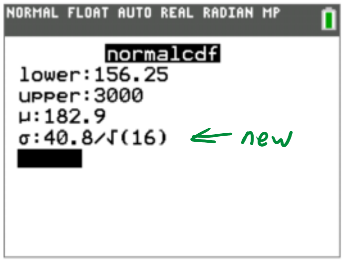


Thus, $$P(\bar{x} > 156.25) = 1 - 0.0045$$

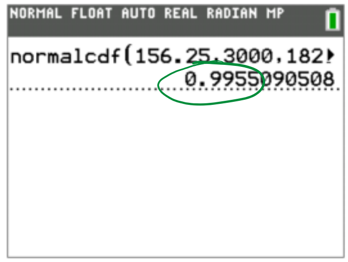
$$= \boxed{0.9955}$$

TI-84: Change σ to $\sigma/\sqrt{n}$



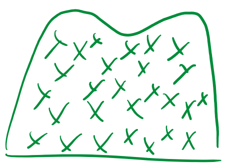


← new



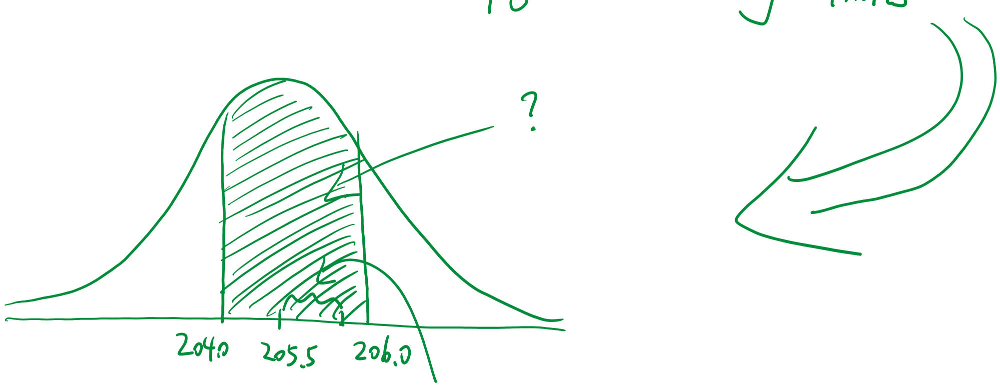
eg Given that the overhead reach distances of adult females: μ = 205.5 cm, σ = 8.6 cm, and overhead reach distances are normally distributed. The overhead reach distances are used in planning assembly work stations.
If 40 adult females are randomly selected, find the probability that they have a mean overhead reach between 204.0 cm and 206.0 cm.

S:



$$\frac{x_1 + x_2 + \cdots + x_{40}}{40}$$

many times



$$\frac{8.6}{\sqrt{40}}$$