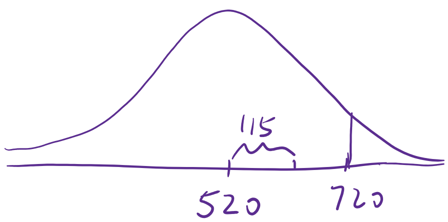
II. Cont.

67. During a certain year, 1,475,623 students heading to college took the SAT. The distribution of scores in the math section of the SAT follows a normal distribution with mean $\mu = 520$ and standard deviation $\sigma = 115$.

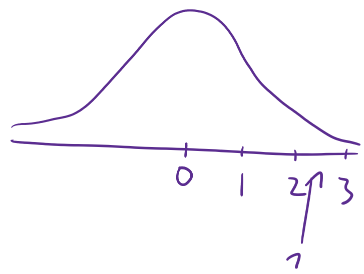
a. Calculate the *z*-score for an SAT score of 720. Interpret it using a complete sentence.
b. What math SAT score is 1.5 standard deviations above the mean? What can you say about this SAT score?
c. During a different year, the SAT math test had a mean of 514 and standard deviation 117. The ACT math test is an alternate to the SAT and is approximately normally distributed with mean 21 and standard deviation 5.3. If one person took the SAT math test and scored 700 and a second person took the ACT math test and scored 30, who did better with respect to the test they took?

S: a.

$$z = \frac{x - \mu}{\sigma}$$

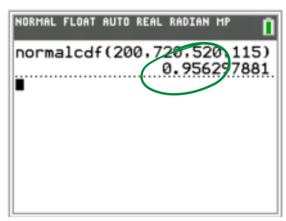


$\Rightarrow$



$X = 720$, $z = \frac{720 - 520}{115}$

$\approx \boxed{1.74}$ $\rightarrow$



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S: b. 1.5 means $z = 1.5$, the SAT score is:

$$z = \frac{x - \mu}{\sigma}$$

$$1.5 = \frac{x - 520}{115}$$

$\leftarrow$ solve for x

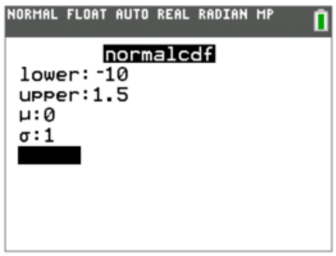
$$1.5 \cdot 115 = x - 520$$

$$173 = x - 520$$

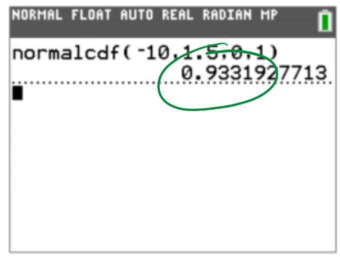
$$+520 \qquad\qquad +520$$

$$\boxed{693} = x$$

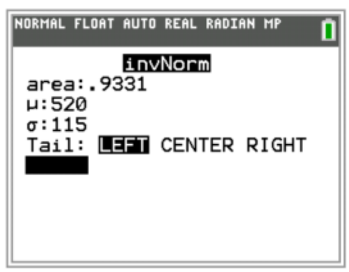
$z = 1.5 \rightarrow$



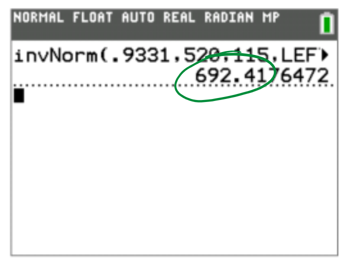
$\rightarrow$



then $p = 0.9331 \rightarrow$



$\rightarrow$



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S: c. SAT vs ACT :

$$z_{SAT} = \frac{x - \mu}{\sigma}, \qquad z_{ACT} = \frac{x - \mu}{\sigma}$$

$$= \frac{700 - 514}{117} \qquad\qquad = \frac{30 - 21}{5.3}$$

≈ 1.59 ≈ 1.70 ✓

since $z_{ACT} = 1.70 > z_{SAT} = 1.59 \Rightarrow$ ACT has more probability than SAT

$\boxed{z_{ACT} \text{ did better.}}$

78. In a certain presidential election, Alaska's 40 election districts averaged 1,956.8 votes per district for Candidate A. The standard deviation was 572.3. (There are only 40 election districts in Alaska.) The distribution of the votes per district for Candidate A was bell-shaped. Let X = number of votes for Candidate A for an election district.

a. State the approximate distribution of X.
b. Is 1,956.8 a population mean or a sample mean? How do you know?
c. Find the probability that a randomly selected district had fewer than 1,600 votes for Candidate A. Sketch the graph and write the probability statement.
d. Find the probability that a randomly selected district had between 1,800 and 2,000 votes for Candidate A.
e. Find the third quartile for votes for Candidate A.

S: a. $\boxed{X \sim N(1956.8,\ 572.3)}$

b. $\boxed{\text{Population, because all districts are included.}}$

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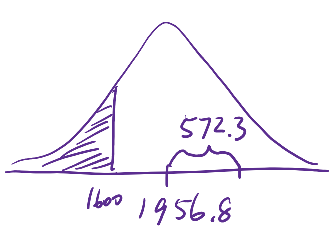
c.

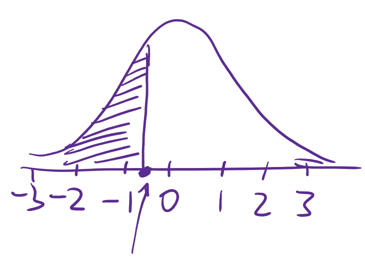


$\rightarrow z = \dfrac{x - \mu}{\sigma}$

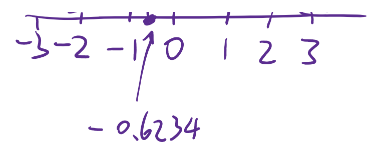
$= \dfrac{1600 - 1956.8}{572.3}$

≈ -0.6234

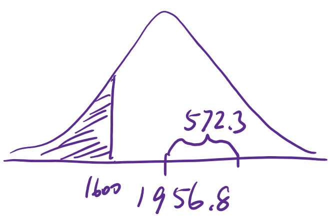
$\rightarrow$



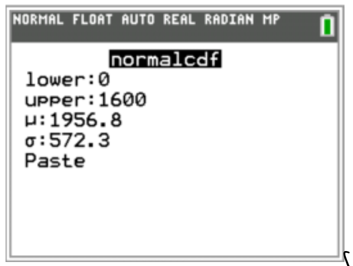
$$\approx -0.6234$$



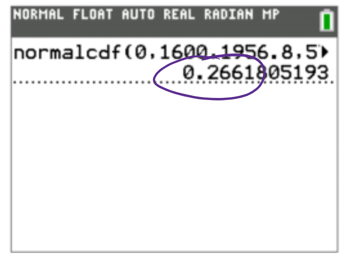
$\rightarrow$ Table $\rightarrow$ $P = \boxed{0.2676}$



$\rightarrow$



$\rightarrow$



$P = \boxed{0.2662}$ $\leftarrow$ This is more accurate.

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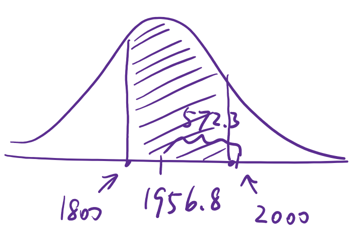
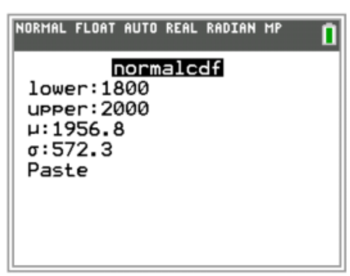
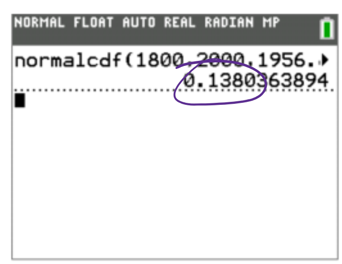
S: d.



$\rightarrow$



$\rightarrow$



$P = \boxed{0.1380}$

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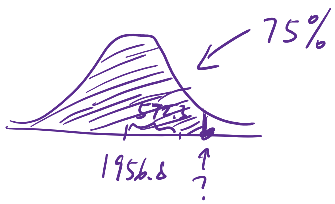
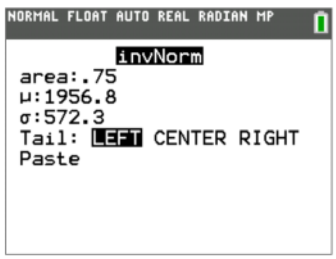
e. Find the third quartile for votes for Candidate A.

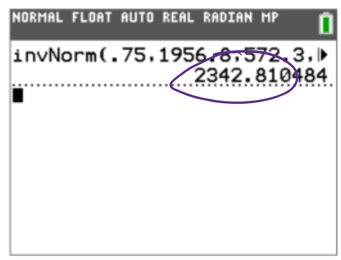
S: e. Third quartile = 75th $\Rightarrow$ $p = 0.75$, from 0%



$p = 0.75$ $\rightarrow$



$\rightarrow$



Thus, 75th $\approx$ $\boxed{2342.8}$ votes