VIII. Binomial Distribution ← TI-84 is needed

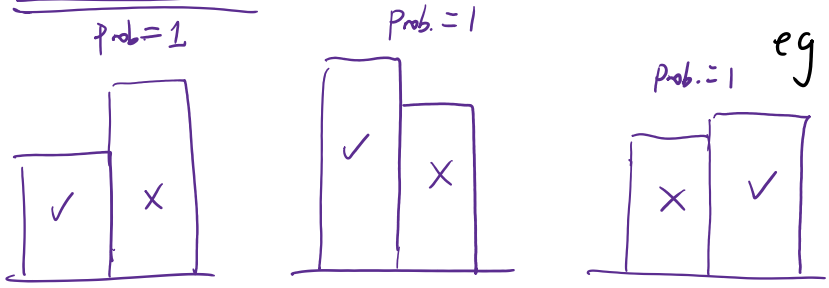
It is about probability occurs randomly with certainty.

It is to find one of the two outcome's probability.

Requirement:

- A fixed number of trials ← n
- Independent; random variable ← 'very hard' to human
- Two categories ← (Yes or no)
  - eg know an app: Vine, flickr
  - eg graduated
  - eg survive or die
  - eg voters for 2 parties
  - ⋮

2 cases:



Notation: We only have two outcomes, which is 'success' or 'failure'

x is the variable from 0 to n, inclusively. ← 0 is there, don't forget 0.

n is the total number of trials.

$P(x)$ is the probability of x.

p is the probability of 'success' ← only for formula

p is the probability of 'success'

q is the probability of 'failure'. $q = 1 - p$.

← only for formula

↙ no use in 2025.

Calculation: 3 ways

1. Table
2. Formula: $P(x) = \frac{n!}{(n-x)!\,x!} p^x q^{n-x}$
3. TI-84 ← we use this

eg In 2019, given there is a 0.85 probability that any given adult knows of Twitter, use the binomial probability formula to find the probability of getting exactly three adults who know of Twitter when five adults are randomly selected.

S: $p = 0.85$, $q = 1 - p = 1 - 0.85 = 0.15$, $x = 3$, $n = 5$

ask for $P(3)$

$$P(x) = \frac{n!}{(n-x)!\,x!} p^x q^{n-x}$$

$$P(3) = \frac{5!}{(5-3)!\,3!} 0.85^3 0.15^{5-3}$$

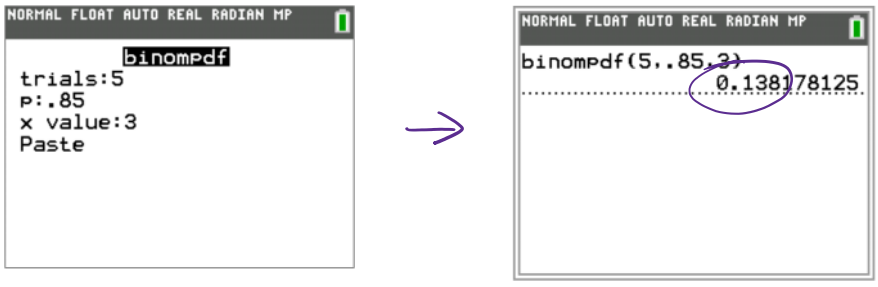
$$\approx \boxed{0.14}$$

$\frac{5!}{2!3!} = \frac{5 \cdot 4 \cdot 3!}{2 \cdot 1 \cdot 3!} = 10$

TI-84: 2nd → distr → A: binopdf(

↑ particular ← for "exactly"

particular ← for exactly



Eg. Assume that randomly guesses are made for 8 multiple choice questions on an SAT test, so that the probability of success p = 0.2. Find the probability for:

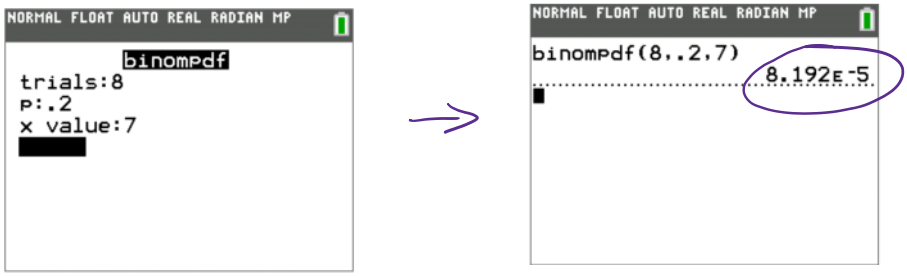
a. the number of x of correct answer is exactly 7. ← ask for P(7)

b. the number of x of correct answer is fewer than 5.

c. the number of x of correct answer is at least 2.

S: a. $p = 0.2$, $n = 8$, $x = 7$

$P(7) =$



0.00008192

5 decimal places

b. fewer than 5: 0, 1, 2, 3, 4, ~~5~~ ← do not forget

It is $P(0) + P(1) + P(2) + P(3) + P(4)$

⇒ up to 4, inclusively

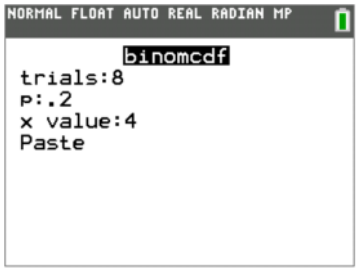
B: binocdf(

↑ cumulative ← up to

↑ cumulative ⟵ up to

$P(0) + P(1) + P(2) + P(3) + P(4)$

$=$



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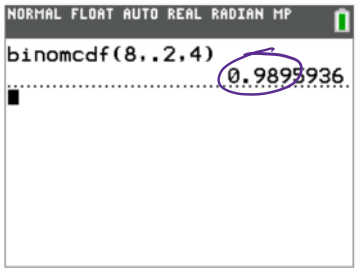


Eg. Assume that randomly guesses are made for 8 multiple choice questions on an SAT test, so that the probability of success p = 0.2. Find the probability for:

a. the number of x of correct answer is exactly 7. ✓

b. the number of x of correct answer is fewer than 5. ✓

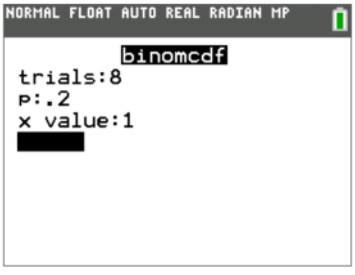
c. the number of x of correct answer is at least 2.

S: c. At least 2: 2 or more: (include) 2, 3, 4, 5, 6, 7, 8, 9, 10...

It is $P(2) + P(3) + P(4) + P(5) + P(6) + P(7) + P(8)$

$= 1 - (P(0) + P(1))$

up to 1 →



$= 1 - 0.503$ ⟵

$= \boxed{0.497}$

eg

Assume that when human resource managers are randomly selected, 41% say job applicants should follow up within two weeks. If 5 human resource managers are randomly selected, find the probability that at least 2 of them say job applicants should follow up within two weeks.

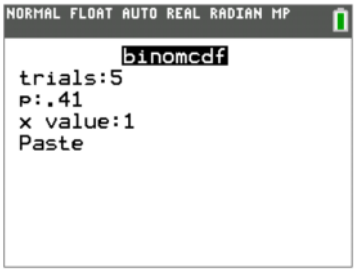
S:

$p = 0.41$, $n = 5$, ask $P(\text{at least } 2)$

$P(\text{at least } 2) = P(2) + P(3) + P(4) + P(5)$

$= 1 - (P(0) + P(1))$

$= 1 - \text{'binocdf 1'}$



$= 1 - 0.32$

$= \boxed{0.68}$

ii. Parameters

$\boxed{\text{Mean}: \mu = np}$

Variance: $\sigma^2 = npq$

$\boxed{\text{Standard deviation}: \sigma = \sqrt{npq}}$

Max/Min cutoff:

$\boxed{\begin{array}{l}\text{Max usual cutoff}: \mu + 2\sigma \\ \text{Min usual cutoff}: \mu - 2\sigma\end{array}}$

← real life 2 std. dev. away is considered special.

eg McDonald's has a 95% recognition rate. A special focus group consists of 12 randomly selected adults. For such a group, find the mean and standard deviation.

S: $p = 0.95$, $q = 1 - p = 1 - 0.95 = 0.05$, $n = 12$

mean: $\mu = np = 12 \cdot 0.95 = \boxed{11.4}$ people

std. dev.: $\sigma = \sqrt{npq} = \sqrt{12 \cdot 0.95 \cdot 0.05} \approx \boxed{0.75}$ people

eg McDonald's has a 95% recognition rate. A special focus group consists of 12 randomly selected adults. Use the range rule of thumb to find the maximum and minimum usual number of people who would recognize McDonald's.

S: $\mu = 11.4$, $\sigma = 0.75$ from above

Max: $\mu + 2\sigma = 11.4 + 2 \cdot 0.75 = \boxed{12.9}$ people

Min: $\mu - 2\sigma = 11.4 - 2 \cdot 0.75 = \boxed{9.9}$ people

meaning: If 9 people recognize, then it is weird.