VI. Prob. Dist.'s Parameters

i. mean

$$\boxed{\mu = \sum [x\,P(x)]}$$

$\sum x_i \cdot P(x_i)$

$x_1 P(x_1) + x_2 P(x_2) + \cdots x_n P(x_n)$

The mean in a probability distribution is also called expected value.

ii. Standard deviation

Variance: $\sigma^2 = \sum (x-\mu)^2 P(x)$

std. dev.: $\sigma = \sqrt{\sum (x-\mu)^2 P(x)}$

eg. The following describe the probability distribution for the number of girls in two births. Find the mean and the standard deviation.

x	$P(x)$
0	0.25
1	0.5
2	0.25

S: mean: $\mu = \sum x P(x) = 0 \cdot 0.25 + 1 \cdot 0.5 + 2 \cdot 0.25$

$= 0 + 0.5 + 0.5$

$= \boxed{1}$ girl

Standard deviation: $\sigma = \sqrt{\Sigma (x-\mu)^2 P(x)}$

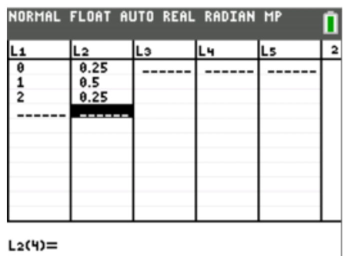
$$= \sqrt{(0-1)^2 \cdot 0.25 + (1-1)^2 \cdot 0.5 + (2-1)^2 \cdot 0.25}$$

$$= \sqrt{1 \cdot 0.25 + 0 \cdot 0.5 + 1 \cdot 0.25}$$

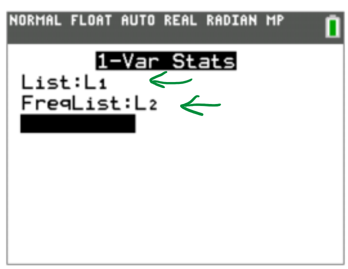
$$= \sqrt{0.5}$$

$$\approx \boxed{0.71} \text{ girl}$$

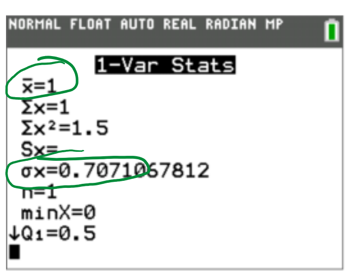
TI-84: Put on to two lists:



Stat → CALC → 1: 1-Var Stats:



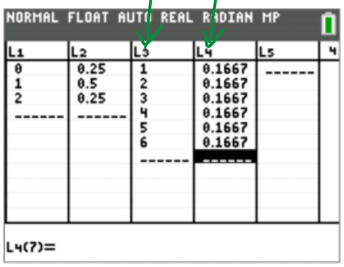
⇒



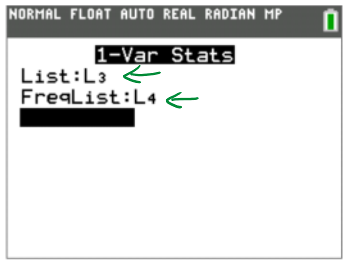
eg Find the mean of the number of spots that appear when a die is tossed.

x	1	2	3	4	5	6
$P(x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

S:



→



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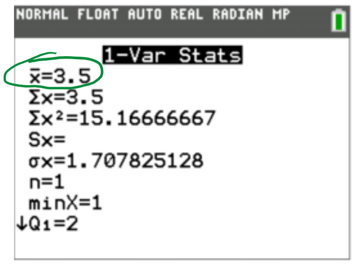
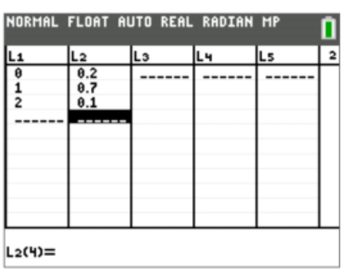


eg Find the mean and standard deviation for the following:

x	0	1	2
$P(x)$	0.2	0.7	0.1

S:



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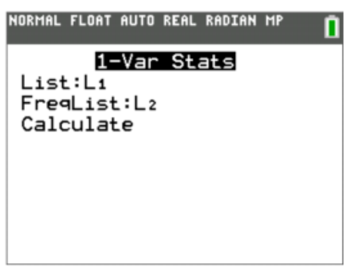

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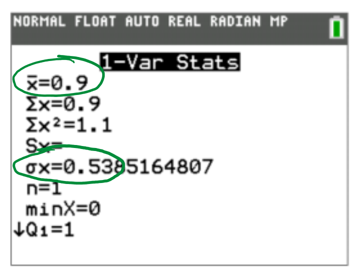

$E(x)$, E.V., mean

VII. Expected Value

the mean μ for '2 items'

The expected value is the average gain or loss of an event in a procedure.

in a procedure.

$$\boxed{E(x) = \sum x\,p(x)}$$ usually for 2 distributions

Property: If $E(x) > 0$, it is gain.

If $E(x) < 0$, it is loss.

eg Real life:

Time	Activity
10 hrs	\$
7 hrs	rest

$$E.V. = 10 \cdot \$ + 7 \cdot \text{rest}$$

$$= \boxed{\text{Daily life}}$$ ← result

Eg. Find the expect value of the procedure below.

Bet	Earn
1	-1
2	-3

S: $E.V. = 1 \cdot (-1) + 2 \cdot (-3)$

$= -1 + -6$

$= \boxed{-7}$

eg Suppose you play a game with a biased coin. You play each game by tossing the coin once. P(heads) = $\frac{2}{3}$ and P(tails) = $\frac{1}{3}$. If you toss a head, you pay \$6. If you toss a tail, you win \$10. What is the expected value?

S:

Money	Prob
$-\$6$	$\frac{2}{3}$
$\$10$	$\frac{1}{3}$

$$E.V. = -6 \cdot \frac{2}{3} + 10 \cdot \frac{1}{3}$$
$$= -4 + \frac{10}{3}$$
$$\approx \boxed{-0.67} \text{ dollars}$$

eg Suppose you play a game of chance in which five numbers are chosen from 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. A computer randomly selects five numbers from zero to nine with replacement. You pay \$2 to play and could profit \$100,000 if you match all five numbers in order (you get your \$2 back plus \$100,000). Over the long term, what is your expected profit of playing the game?

S:

$\frac{1}{10}$ for each number, with replacement

five number: $\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} = 0.00001$

	Money	Prob
win →	\$100,000	0.00001
lose →	$-\$2$	0.99999

Prob. of win: 0.00001

Prob of lose: $1 - 0.00001 = 0.99999$

$$E.V. = 100000 \cdot 0.00001 + -2 \cdot 0.99999$$

$\approx \boxed{-1}$ dollar

Eg. An insurance company estimates the probability of an earthquake in the next year to be 0.0013. The average damage done by an earthquake it estimates to be $60,000. If the company offers earthquake insurance for $100, what is their expected value of the policy?

S:

not free →

Money	Probability
\$60,000 − \$100	0.0013
−\$100	1 − 0.0013

←

Earthquake prob. = 0.0013

No earthquake prob. = 1 − 0.0013

⇒

Money	Probability
59900	0.0013
−100	0.9987

$$E.V. = 59900 \cdot 0.0013 + -100 \cdot 0.9987$$

$$= \boxed{-22} \text{ dollars}$$

eg A roulette has 18 green trays, 18 red trays, and 1 white tray for the ball to land in. The casino takes your bet of $5 that the ball will land in a green tray. The casino will pay you $10 if the ball lands on the color green. The probability of winning by betting on green is 18/37. What is the expected value for the bettor?

S:

$18 + 18 + 1 = 37$

\$10 − \$5 →

Money	Probability
\$5	$\frac{18}{37}$
−\$5	$\frac{19}{37}$

tcsjohnhuxley
Custom Roulette Wheel ...

Northern Quest Resort & Casino
Live Roulette Tables - Northern Quest ...

Casino.com
European Roulette Whe...

win: $\frac{18}{37}$

lose: $1 - \frac{18}{37} = \frac{19}{37}$

$-\$5 \mid \frac{19}{37}$

lose: $1 - \frac{18}{37} = \frac{19}{37}$

$$E.V. = 5 \cdot \frac{18}{37} + -5 \cdot \frac{19}{37}$$

$$\approx \boxed{-\$0.14}$$

eg Now suppose you bet $1 on #12. If ball lands on #12, you get $35. Otherwise you lose $1. What is the new expected value going to be?

S:

Total is 37

Only one

$\$35 - \$1 \rightarrow$

Money	Probability
$34	$\frac{1}{37}$ ← get #12
-$1	$\frac{36}{37}$ ← not get #12 : $1 - \frac{1}{37} = \frac{36}{37}$

$$E.V. = 34 \cdot \frac{1}{37} + -1 \cdot \frac{36}{37}$$

$$\approx \boxed{-\$0.05}$$