I. Probability ← chance that something may happen

$0 \le P \le 1$

i. Defn

The probability is the calculation of events occur over the overall procedure of an experiment.

- <u>simple event</u> is an outcome of a procedure. (your outcome that you find.)
- <u>sample space</u> is consisting of all possible simple events. It contains of all outcomes that we can not break down any further.

Notation: A, B and C are events,

$Pr(A)$ – the probability of event A.

$Pr(B)$ – . . . B.

$Pr(C)$ – . . . C

$Pr(\text{phrase})$ – . . . phrase

$\boxed{\text{eg } P(\text{math major})}$

rule

Prob. = 1

⋮

0.95 ← cutoff for very high

⋮

0.05 ← cutoff

0.05 ← cutoff for very low

Prob. = 0

* Remember: 0.95 is the cutoff for very high probability.

0.05 is the cutoff for very low probability.

Overall,

$$\boxed{\begin{gathered} Pr(A) = \frac{\text{num of times occurred}}{\text{total num of times}} \\ \underset{(0\%)}{0} \le P(A) \le \underset{(100\%)}{1} \end{gathered}}$$

≈ 1

eg What event(s) has a very very high probability?

- the sun will come out tomorrow
- lightening in a thunder storm
- you are learning
- you alert your Exam 1's mistake

⋮

≈ 0

eg What event(s) has a very very low probability?

[illegible] has a very very low probability ?

- winning a Jackpot in a lottery.
- Snow in August here in Turlock.
- Commerial airplane accidents

⋮

*. Now, when can't calculate the probability, but it is a possibility that may happen (it can not be predicted) is called a random variable. The process is called the random process.

- health issue
- weather
- earthquake
- games winning

⋮

all of these has probabilities, but they are random variables

eg

Procedure	Events	Sample Space
birth	'1 girl'	2^n
1	.	$2^1 = \{b, g\}$
2	.	$2^2 = \{bb, bg, gb, gg\}$

3 . $2^3 = \{bbb, bbg, bgb, bgg, gbb, gbg, ggb, ggg\}$

4 . $2^4 = \{\underbrace{bbbb, \ldots ggg g}_{16 \text{ of them}}\}$

⋮ ⋮ ⋮

eg When two children are born, find the probability of getting the children of the same gender.

S: Sample space $= \{bb, bg, gb, gg\}$

$$Pr(\text{same gender}) = \frac{\text{'bb, gg'}}{4}$$

$$= \frac{2}{4}$$

$$= \boxed{\frac{1}{2}}$$

eg When three children are born, find the probability of getting the children of the same gender.

S: Sample space $= \{\overset{\checkmark}{bbb}, bbg, bgb, bgg, gbb, gbg, ggb, \overset{\checkmark}{ggg}\}$

$$Pr(\text{same gender}) = \frac{\text{'bbb, ggg'}}{8}$$

$$= \frac{2}{8}$$

$$= \boxed{\frac{1}{4}} \text{ or } \boxed{0.25}$$

eg When three children are born, find the probability of having exactly one boy.

S: Sample space = {bbb, bbg, bgb, bgg, gbb, gbg, ggb, ggg}

3 2 2 1 2 1 1 0

$Pr(\text{one boy}) = \frac{\text{'bgg, gbg, ggb'}}{8}$

$= \boxed{\frac{3}{8}}$ or $\boxed{0.375}$

eg When three children are born, find the probability of getting at least one girl.

S: Sample space = {bbb, bbg, bgb, bgg, gbb, gbg, ggb, ggg}

1 1 2 1 2 2 3

At least one : one or more — 1 ✓, 2 ✓, 3 ✓

P(at least one girl) = P(one or more girls)

$= \boxed{\frac{7}{8}}$ or $\boxed{0.875}$

ii. Complement ← "the other pair"

The complement of event A, denotes $\bar{A}$, with $Pr(\bar{A})$ for probability.
The $Pr(\bar{A})$ is the probability of event A that does not occur.

$$\boxed{Pr(\bar{A}) = 1 - Pr(A)}$$

eg 1010 U.S. adults were surveyed and 202 of them were smokers.

Let A = smokers

Then, $Pr(A) = \frac{202}{1010} = 0.2$ ← Prob. of smokers

Now, $P(\text{not a smoker}) = Pr(\bar{A})$

$= 1 - Pr(A)$

$= 1 - 0.2$

$= \boxed{0.8}$

eg When five children are born, find the probability of not getting the children of the same gender.

S:

Sample Space = {bbbbb, bbbbg, . . . ggggg} ← $2^5 = 32$ of them

Let S be the same gender, then

$Pr(S) = \frac{\text{'bbbbb, ggggg'}}{32} = \frac{2}{32}$

$Pr(\bar{S}) = 1 - P(S)$

$= 1 - \frac{2}{32}$

$= \frac{30}{32}$

$$= \boxed{\frac{15}{16}} \text{ or } \boxed{0.9375}$$